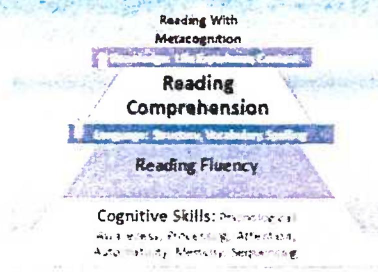
Reading Comprehension



Explicitly Teaching Inferences

1. Based on the facts on page ____, what conclusion can you make?

2. Why is it important that _______________?

3 After looking at the picture on page ___, what can you know about a ______?

Explicitly Teaching Critical Literacy

1. What was the author's purpose in writing this story?

2. Where can you find the author's purpose? (It is always the main idea in the first paragraph on the first page)

3. Why is writing about this topic important?

4. Was the story difficult to understand? Why or why not?

Explicitly Teaching Creative Literacy

1. Can you tell me about a time when _________________?

2. Can you tell me about a _____________ you have seen?

3. If you were going to write this story how would you chose to end it? Why would you chose that ending?

4. What other information could the author have given the audience? Why would that information be important to know?

Teaching Visual Literacy Skills

Visual literacy is the ability to evaluate, apply, or create conceptual visual representations. Skills include the evaluation of advantages and disadvantages of visual representations, to improve shortcomings, to use them to create and communicate knowledge, or to devise new ways of representing insights.

Reading A-Z

FLUENCY STANDARDS TABLE

Recommended reading rates, or words read per minute, for grades one through six were examined from three separate research studies. The findings of these studies were used by Reading A-Z to establish an average early and end reading rate per grade level. Your student's reading rates can be compared to these average rates as a way to determine whether they are making progress in their ability to recognize words automatically. The comparison can also be used to determine whether a student's reading rate is near the grade level standard. For example, a beginning third grade student with a reading rate of 110 WPM can be considered on level. However, a third grade student with a reading rate of 60 WPM is recognizing words at a rate similar to a first grader and will likely need additional instructional support to increase his or her reading rate.

READING A-Z RECOMMENDATIONS
WORDS PER MINUTE (WPM)

GRADE	BEGINNING RATE	MID-YEAR RATE	END RATE
1	50	60	70
2	70	80	100
3	100	120	130
4	130	135	140
5	140	150	160
6	160	165	170

Grade 8
Technical Document

Definition of Technical Writing

At the eighth grade, the California English-Language Arts Content Standards call for students to be able to write clear, coherent, and sequentially organized technical documents which explain a complex operation or situation.

Importance

The ability to read and write technical documents is crucial in today's technology-driven society. In order to successfully operate such machines as videocassette recorders, computers, telephone answering machines, or even basic kitchen appliances, one must first consult the technical guides which accompany them. Technical writing is a type of informational writing, a writing domain with which students need to be familiar and proficient if they are to succeed in school and the world at large.

Prior Instruction for Writing Technical Documents

The prior instruction necessary for students to meet the grade level standards in writing requires the implementation of a balanced writing program. This includes daily whole-class demonstrations and instruction (writing aloud, shared writing, interactive writing), frequent individual instruction (guided writing), and daily opportunities to write independently.

Below are suggestions for classroom activities designed to prepare students to meet writing standards, especially those related to writing technical documents.

Analyze Style and Format of Technical Documents

Students will better understand the requirements of this assignment if they are first given the opportunity to study real-life sources of functional documents for formatting and organization. Technical instructional materials for operating video-cassette recorders, calculators, or other everyday tools are easy to obtain from school resources and staff. Content area textbooks are also helpful as models for this part of the activity. The teacher will need to point out the following:

- Some technical documents are written in narrative style
- Others are in an abbreviated format (incomplete sentences, bulleted sequencing, charts or graphs, etc.)

The teacher may wish to point out how certain features aid comprehension, such as:

- Spacing
- Boldface
- Italics
- Font styles and sizes
- Color

Write Simple Technical Instructions
Once students are familiar with the various forms of technical documents, let them practice writing simple instructions on their own or in pairs using a format of their own choosing. They should be encouraged to write instructions for an activity that is easily tested in class, such as:

- How to Tie a Shoe
- How to Refill a Mechanical Pencil
- How to Staple Papers Together

Students may then test their instructions on other classmates to see how well the writer sequenced the steps involved and what sorts of gaps, if any, exist in the step-by-step directions.

Analyze Organizational Features of Technical Writing
The same functional documents used in the above activity may also be used to demonstrate how technical writing is organized. Even though such writing does not fall into the category of an essay, it includes many of the same components:

- a clear introduction to the topic
- a body section which further describes that topic
- a conclusion that provides clear and satisfying closure
- transitional vocabulary to tie the sections together.

Provide time for students to work collaboratively to discover the organization of various technical documents.

Analyze and Apply Sequencing Vocabulary and Spatial Details
Allow students ample time to collaboratively analyze various functional documents for such sequencing words as *first, next, then,* and *finally.* Ask them to point out relevant spatial vocabulary which helps orient the reader, such as *top, bottom, front, back, left, right,* as well as descriptions regarding width, height, length, depth, diameter, or circumference. Descriptive vocabulary should also be noted, such as *for instance, for example, such as, in addition, also,* and *another.*

After students have become familiar with these components, allow them to work in pairs to practice giving oral step-by-step instructions in the activity below:

Hidden Design Recreation
Each student creates a simple design out of geometric objects (either by drawing or arranging cut-out pieces) which is kept hidden from the partner. Once the initial design is created, the student provides oral instructions to the partner (who has access to the same materials) which, if accurate, will lead to a recreation of the exact same design. Students should quickly realize how sequential and spatial details add greatly to the successful completion of this exercise.

When completed, ask students to reflect on the exercise, considering information which helped or hindered in completing the assignment (e.g. precise vocabulary, sequencing of instructions, clear description, etc.).

Note: It may be helpful to list these "helpful hints" on chart paper for future reference.

Edit for Grammar, Punctuation, Capitalization, and Spelling

Regardless of the narrative or non-narrative writing style of a technical document, students may analyze sentence structure, punctuation, capitalization, spelling, and grammar. Special attention should be given to complex as opposed to simple sentence structures. Also, students should note how sentence variety contributes to the readability of the piece.

Students should practice editing and revising short passages (4 - 5 sentences), preferably related to informative text such as that found in technical documents. Teachers may prepare (or may allow students to pair up to create) short passages, to be used as mini-lessons, with a few intentional errors in grammar, punctuation, capitalization, and spelling. After modeling the editing process, the teacher allows students to practice finding the errors, comparing the type and number found among students to determine further instruction.

Combine Sentences for Variety and Cohesiveness

Revising a number of simple sentences into one combined or complex sentence is another excellent activity. For example, students are given the following four simple sentences to combine:

1. Pick up the receiver.
2. Listen for the dial tone.
3. Press the numbered buttons to dial the telephone number.
4. Dial the telephone number you desire.

Revision Examples:

1. Pick up the receiver, listen for the dial tone, and press the numbered buttons to dial the telephone number you desire.
2. After picking up the receiver and listening for the dial tone, press the numbered buttons to dial your desired telephone number.

Students enjoy making up their own simple sentences for combining practice, as well as breaking complex sentences into their simple sentence parts. Point out the improvement in clarity, smoothness, and variety when sentences are combined.

Directions for the Writing Assessment "How to Operate a Tool"

To the Teacher

You are encouraged to treat this prompt as a series of class lessons, even though the student work produced may be used to determine if the student has met state standards. These directions provide guidelines, but pease use your discretion in walking students through the prompt. If you plan to use the student writing to determine whether the student has met, in part, the grade level standards in writing, then you should conduct the following as consistently as possible throughout the year.

Once prior instruction has taken place, the three-session assessment process begins. The number of class days involved will vary according to individual teaching situations and preferences. However, if this assessment is being used across the school site or district at this grade level, this process should also be as consistent as possible.

In this assessment document, students are asked to identify the sequence of activities needed to operate a tool or piece of equipment they regularly use in school. They must include all factors and variables that need to be considered in operating this tool and employ formatting techniques, such as headings, bullets, and underlining, to aid comprehension.

General Guidelines for Assessing Students

In order to maintain consistency, the following guidelines may be useful:

- Use the same prewriting activities for each trial.
- Follow the directions at each step.
- Do not provide answers to student questions that would directly meet the standards.
- Students may use spelling resources which are regularly available in the classroom (wall charts, word lists, dictionaries, thesaurus). Students may **not** use computers or electronic spelling aids.
- Do not allow peer or teacher assistance during the actual writing process.
- Do not allow papers to be taken home during the assessment process.

The following may be adjusted to meet student needs:

- Rephrase the directions for better student understanding.
- Allow students access to their primary language if that will assist in understanding the task.

Materials

Materials included:

- Teacher Instructions
- Student Writing Prompt
- Student Checklist
- Prewriting Graphic Organizers
- Teacher Scoring Guide

To be provided by the teacher and/or student:

- Writing paper
- Writing utensils
- Dictionaries, thesaurus, and other resources regularly used in the classroom

Time Limits

Three sessions are required for the assessment portion of this lesson. These sessions may take place over three days or less, depending on site and teaching considerations. Parts I and II should last no more than 60 minutes each. Part III should last no more than 120 minutes.

Introducing the Assessment

Part I: Prewriting (60 minutes)

The purpose of the prewriting activity is to connect the activities included in the prior instruction component to the actual writing students will do. This portion of the assessment allows students the opportunity to organize their ideas into properly sequenced and formatted technical documents which are clear, coherent, and accurate by following three steps: model, practice, and apply.

Model:
Using the prewriting graphic organizer below (blackline master included in Grade 8 Appendix), introduce the prompt, discussing as needed for understanding and generating ideas.

- List topic ideas on the board or chart paper.
- Explain the purpose of using the graphic organizer.
- Model the appropriate steps involved in completing the chart.
- Use an example that is unlikely to be used by students (for instance, creating step-by-step instructions for using a common tool **not** usually used in a classroom, such as a toothbrush or washing machine).

Prewriting Graphic Organizer
"How to Operate a Tool"

Directions for Operating a:
________________________ (name of tool)

Importance/function of tool:

Information needed before using tool:

OPERATIONS FOR PROPER USE:
•
•
•
•
•

Helpful Hints:
•
•
•

Practice:
Once students have seen how to complete the organizer, pass out clean individual copies. Working individually or collaboratively, each student completes his/her own organizer and checks with a partner for clarity, sequencing and accuracy of ideas.

Apply:
Students revise their graphic organizers as needed to begin transferring that information to the first draft of their technical document piece.

Collect all student papers at the end of Part 1.

Part II: Writing (60 minutes)

When students are ready to begin their first drafts of the writing prompt, the following steps should be followed:

- Pass out collected papers from Part I.
- Review prewriting ideas as necessary and/or desired.
- Review the writing prompt.
- Explain and clarify the student checklist .
- Students write first drafts individually—no outside help is allowed at this point.
- If time allows, students may begin editing and revising their own drafts using dictionaries, thesaurus, other regular classroom resources, and the checklist as guides.
- Collect all student papers at the end of Part II.

Grade 8 Writing Prompt
"How to Operate a Tool"

Writing Situation:
There are many different types of tools or equipment you use daily in school. These tools help you perform and learn in many of your classes. For example, you might use a calculator in math, a dictionary in English, a microscope in science, or a computer search engine in the library.

Writing Directions:
Write instructions explaining how to use a tool or piece of equipment which you regularly use in one of your classes. Identify the tool or piece of equipment, explain its function and/or importance, and provide step-by-step instructions which specifically describe how to use this item. Be sure to clearly explain each step so that your fellow classmates could successfully use this piece of equipment just by following your instructions, even if they have never used it before.

Grade 8 Student Checklist
"How to Use a Tool"

Please check the draft of the essay for the following items:

Writing Application Technical Document	Yes	No.	Comments
•Identifies the equipment/tool •Discusses importance and/or function of tool •Includes information needed before, during, and after using tool •Discusses using tool safely and effectively (e.g. cautions, helpful hints) •Describes sequence of activities necessary to operate the tool •Uses proper formatting techniques (e.g. headings, bullets, underlining)			
Writing Strategies			
•Uses effective transitions between sentences and paragraphs •Uses appropriate organization (introduction, body, conclusion) •Includes spatial details (e.g. top/bottom, left/right, width/length/heighth) •Includes precise, descriptive vocabulary •Provides detailed step-by-step instructions			
Writing Conventions			
•Varies sentence structure (simple, compound, complex) •Uses correct punctuation •Uses correct capitalization •Spells correctly			

Part III: Editing/Revision and Final Draft (120 minutes)

This final portion of the assessment allows students the time and opportunity to improve their drafts before writing the final pieces. Final drafts will be assessed according to the content standards criteria presented on the scoring guide. This same criteria is outlined on the student checklist which will be used during this session to focus attention on areas in need of improvement. The steps for completing this portion are:

- Pass out collected papers from Session II.
- Students edit and revise their first drafts using allowable classroom resources and checklist.
- If desired, peer response sessions may also be conducted using this checklist. In that case, clean copies will need to be supplied for each student.
- Students write final drafts legibly using dark ink.
- When final drafts are complete, students assemble all materials used in the three-session process, stapling final drafts on top, and submit to the teacher.

Scoring Student Writing

Using the attached four-point scoring guide (blackline master included in Grade 8 Appendix, teachers holistically score student papers in each of three standards-based areas: Writing Applications, Writing Strategies, and Writing Conventions. Students who score a "3" are considered to be at grade level according to this assessment.

ERB
LIGHTING THE PATHWAYS TO LEARNING

WrAP Writing Assessment Portal Program

6+1 Trait® Writing Grades 3-8

Name____________________

Date____________________

T_1 T_2 T_3

OACS
Osceola Adventist Christian School

Organization		Support	
Fluency		Word Choice	
Mechanics		Presentation	
Overall Development		Mode of Cumulative Record	

WRAP Score OACS Score	Organization	Support	Sentence Fluency	Word Choice	Mechanics	Presentation	Overall Development
6 Paper 4	Plan is developed and well followed including the topic, audience, and purpose and an appropriate plan-type. Carefully but subtly organized from beginning to end. Logical order (well sequenced*); Elegant flow of ideas; Provides closure	Supporting details are rich, interesting, and informative throughout; fully developed; Details are relevant and appropriate for the focus	Sentence structures enhance style and effect; Virtually no errors in structure or usage ; Successfully uses more sophisticated, varied sentence patterns	Rich, effective vocabulary throughout; Vivid language; May use figurative language and imagery	Very few or no mechanical errors relative to length or complexity	Presentation shows a pride in the quality of work, all letters are neatly on the line and formed correctly with even spacing, correct slant, and the written presentation is attractive and helps readers understand and remember the information.	Fluent, richly developed; Clear awareness of audience and purpose ; Distinctive, engaging voice; Original, insightful, or imaginative
5 Paper 3	Organized from beginning to end including a plan that is developed with topic, audience, purpose, and plan-type; Logical order (sequenced*); Subtle transitions; Provides closure	Details are strong and varied throughout; Details are relevant and appropriate for the focus	Sentence structures are appropriate to style and effect; Few errors in structure or usage; Moderately successful in using more; sophisticated sentence patterns	Effective vocabulary; Generally successful in using rich language	Few mechanical errors relative to length or complexity	Presentation shows basic neatness with no more than two letters formed and spaced incorrectly, and overall design of the written presentation helps readers understand the information.	Fluent, fully developed; Clear awareness of audience and purpose; Evidence of voice, compositional risks attempted; Cohesive
4 Paper 3	Topic, audience, purpose and plan-type is developed by may not be followed causing minor lapses in order or structure (some breaks in sequencing*); Meaning is subordinate to organizational devices; Contrived transitions; Provides closure	Details are adequate to support the focus; Details are generally relevant to the focus	Some sentence variety; Generally correct structure and usage; Attempts to use more sophisticated sentence patterns	Acceptable vocabulary; Attempts to use rich language; Misuse of bigger grade-level appropriate vocabulary words	Some mechanical errors that do not interfere with communication; Limited text, but mechanically correct	Presentation is readable and basically neat. There are no more than four words spaced incorrectly per line, or four letters per line written incorrectly.	Moderately fluent, adequately developed; Awareness of audience and purpose; Ideas developed but somewhat limited in depth

ERB
LIGHTING THE PATHWAYS TO LEARNING

WrAP Writing Assessment Portal Program

6+1 Trait® Writing

Grades 3-8

Name____________________

Date____________________

T_1 T_2 T_3

OACS
Osceola Adventist Christian School

Page2

WRAP Score OACS Score	Organization	Support	Sentence Fluency	Word Choice	Mechanics	Presentation	Overall Development
3 Paper 2	Lack of planning evident; Poor transitions; andom sequencing*; Attempts closure; Shift in focus	Details lack elaboration; Insufficient relevant details; Important details are omitted	Little sentence variety; Errors in structure or usage interfere with meaning; Over-reliance on simple or repetitive constructions; Chaining; Noticeable errors in usage	Simplistic vocabulary with acceptable but limited word choice; Some errors in word choice	Some mechanical errors that do interfere with communication; Errors are disproportionate to the length or complexity of the piece (errors cause major problems for readers)	60-75% of words, letters, slant, or formation are correct. Presentation is readable, but not particularly neat or of good quality.	Somewhat developed; Some awareness of audience and purpose; Repetitive or too general
2 Paper 1	Lack of planning evident; Thought patterns are difficult to follow; Ideas are not clear or sequenced*; Resembles free-writing, rambling; Continual shifts in focus	Supporting details are listed; Repetitious details; Too few details	No sentence variety; Serious errors in structure or usage; Too brief to demonstrate variety	Simplistic vocabulary with inappropriate and/or incorrect word choice	Noticeable mechanical errors that interfere with communication; Errors are disproportionate to the length or complexity of the piece (errors cause major problems for readers)	About half of the presentation has distracting errors in letter formation, slant, or spacing. The quality of the presentation detracts significantly from readability.	Poorly developed; Poor awareness of audience; or purpose; Ideas and details are not clear
1 Paper 1	Little or no planning; So short or muddled that it lacks organization or focus	Virtually no details; Irrelevant details	Lack of sentence sense; Riddled with errors at the sentence level; Riddled with errors in usage; Too brief to evaluate	Extremely limited vocabulary; Riddled with errors in word choice; Too brief to evaluate	Mechanical errors that seriously interfere with communication; Too brief to evaluate	Letter formation, spacing, slant is imbalanced, cluttered, and shows a lack of pride in the quality of work. The presentation quality interferes with readability.	Not developed; Restates topic; No awareness of audience or purpose; Inappropriate response; Too brief to show development

Student Name ______________________________ **E** **T_1** **T_2** **T_3** **T_4**

School Year 20____ - 20 _____

Teacher ____________________________________

Eighth Grade
Mathematics Common Core Assessment

Number Sense in Functions	_____%
Algebraic Thinking/Operations	_____%
Measurement	_____%
Data/Probability/Statistics	_____%
Geometry	_____%

Assessment for Common Core Mathematics Standards Grade 8

Summary Sheet

Name______________________________ T_1 T_2 T_3

School ____________________________ Year __________

Teacher ____

0 1 2 3 4 __________ % Pre-Algebra

Assessment for Common Core Mathematics Standards Grade 8

Introduction: Summary of Goals

GRADE EIGHT

By the end of grade eight students are expected to have a strong foundation in pre-algebraic skills that will move them into Algebra I their first year of high school. This assessment differs from the assessments in grades K-7. The prior assessments focus on all five strands of mathematics as individual strands, though grades 3-7 combine Measurement and Geometry into one strand. The grade eight assessment encompasses all five strands of mathematics: 1) Number Sense, 2) Operations and Algebraic Thinking, 3) Measurement, 4) Geometry, and 5) Statistics, Data, and Probability into Pre-algebraic Operations. Number Sense is the foundation of algebra, and algebra is the language of Measurement, Geometry, and Statistics, Data and Probability.

Assessment for Common Core Mathematics Standards Grade 8

1. a. Fill in the blanks below with a single appropriate letter to identify each set of numbers with the properties or descriptions of the elements which characterize that set:

The set of:

___ Even Numbers
___ Rational Numbers
___ Irrational Numbers
___ Real Numbers
___ Integers
___ Odd Numbers
___ Natural Numbers
___ Whole Numbers

A. any number equal to a terminating decimal expression
B. {..., -3, -2, -1, 0, 1, 2, 3, ...}
C. any number which is rational or irrational
D. any number of the form $\frac{p}{q}$ where p and q are integers and q is not zero
E. any integer of the form 2k, where k is an integer
F. any integer of the form 2k + 1, where k is an integer
G. any number equal to an infinite decimal expression with no repeating block of digits
H. {0, 1, 2, 3, ...}
I. any number which can be expressed as a ratio
J. {1, 2, 3, ...}

Assessment for Common Core Mathematics Standards Grade 8

1.

[CONTINUED]

b. Which of the following sets of numbers are not closed under addition?

__ The set of real numbers

__ The set of irrational numbers

__ The set of rational numbers

__ The set of positive integers

2.

a. Which number below is the same as $-\frac{2}{3}-(-(-\frac{3}{5}))$?

A. $\frac{4}{15}$ B. $-\frac{4}{15}$ C. $-\frac{1}{2}$ D. $-\frac{19}{15}$

b. What number z satisfies the equation $\frac{2}{3}z = 1$?

c. Calculate and simplify the expression $\sqrt[3]{2\sqrt{16}}$

d. If x = 4, what is $x^{-3/2}\,(x^{100} / x^{99})$?

e. Write $(x^{3/2})^{4/3}\,\frac{\sqrt[4]{x}}{x}$ as x raised to a power

Assessment for Common Core Mathematics Standards Grade 8

3.

a $|2x - 1| = 5$ Find all values of x which make this equation true

b. $3|2 - 5x| + 1 < 10$ Find all values of x which make this inequality true

c. $4|x - 1| = 16$ Find all values of x which make this equation true

d. $|x - 2| > 4$ Find all values for x which make this inequality true

Assessment for Common Core Mathematics Standards Grade 8

4.

a. Simplify

1. $3(2x - 5) + 4(x - 2)$

2. $\frac{1}{2}(6x + 4) - \frac{1}{3}(3 - 6x)$

b. Solve for x

1. $8(x + 1) + 3(2x - 2) = 44$

2. $\frac{1}{3}(12x - 9) - 2(x - 5) \geq 17$

Assessment for Common Core Mathematics Standards Grade 8

5. a. Justify each step below for the solution for x from the equation

$$\frac{2}{3}(x+3)+4(x-8)=2$$

Use the following list:

A. Commutative Property of Addition
B. Associative Property of Addition
C. Commutative Property of Multiplication
D. Associative Property of Multiplication
E. Distributive Property
F. adding the same quantity to both sides of an equation preserves equality
G. multiplying both sides of an equation by the same number preserves equality
H. 0 is the additive identity
I. 1 is the multiplicative identity

To the right of each equation below (and on the following pages) where there is an empty space, write one of the letters 'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', or 'I' to justify how that equation follows from the one above it. For example, the second equation below is justified by 'G' and the third one by 'E'.

Step	**Justification**
$\frac{2}{3}(x+3)+4(x-8)=2$	The given equation
$\frac{3}{2}[\frac{2}{3}(x+3)+4(x-8)]=(\frac{3}{2})2$	G
$(x+3)+6(x-8)=3$	E
$(x+3)+(6x-48)=3$	

5.

[CONTINUED]

Step	Justification
$[(x+3)+6x]-48=3$	
$[x+(3+6x)]-48=3$	
$[x+(6x+3)]-48=3$	
$[(x+6x)+3]-48=3$	
$[(1+6)x+3]-48=3$	
$(7x+3)-48=3$	$1+6=7$
$7x+(3-48)=3$	
$7x-45=3$	$3-48=-45$
$(7x-45)+45=3+45$	
$7x+(-45+45)=48$	
$7x+0=48$	$-45+45=0$
$7x=48$	
$\frac{1}{7}(7x)=\frac{48}{7}$	
$(\frac{1}{7}7)x=\frac{48}{7}$	

5.

[CONTINUED]

Step	**Justification**
$1x = \frac{48}{7}$	$\frac{1}{7} \cdot 7 = 1$
$x = \frac{48}{7}$	

b. The sum of three integers is 66. The second is 2 more than the first, and the third is 4 more than twice the first. What are the integers?

c. During an illness, a patient's body temperature T satisfied the inequality $|T - 98.6| \leq 2$. Find the lowest temperature the patient could have had during the illness.

Assessment for Common Core Mathematics Standards Grade 8

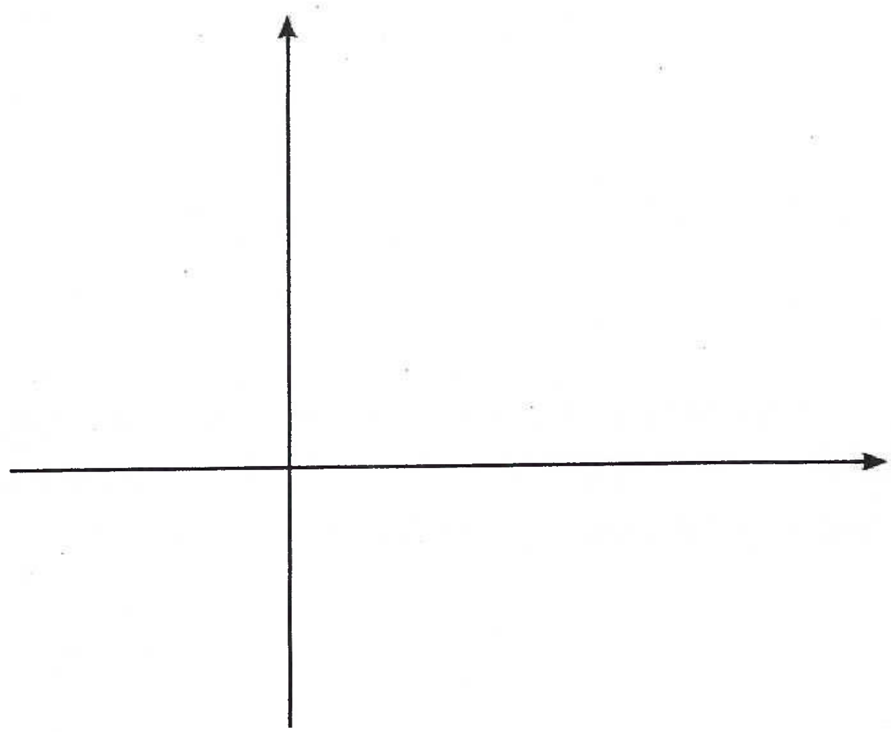
6.

a. Graph the equation: $2x - y = 3$

b. What is the x intercept?

c. What is the y intercept?

d. On your graph, mark the region showing $2x - 3 < y$



Assessment for Common Core Mathematics Standards Grade 8

7.

a. Write an equation involving only numbers that shows that the point $(1\frac{1}{2}, 2)$ lies on the graph of the equation $2y = 6x - 5$.

b. A line has a slope of $\frac{1}{2}$ and passes through the point $(5, 8)$. What is the equation for the line?

a. A line is parallel to the line for the equation: $\frac{1}{2}y = \frac{1}{2}x - 9$. What is the slope of the parallel line?

b. What is the slope of a line perpendicular to the line for the equation $3y = 7 - 6x$?

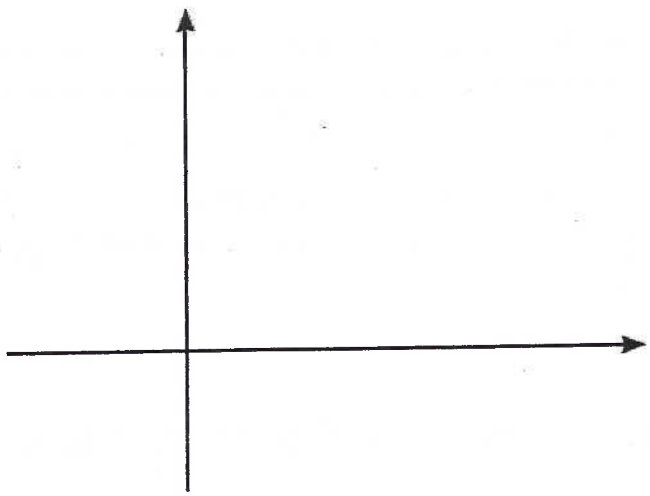
c. What is the equation of a line passing through the point $(7, 4)$ and perpendicular to the line having the equation $3x - 4y - 12 = 0$?

Assessment for Common Core Mathematics Standards Grade 8

9.

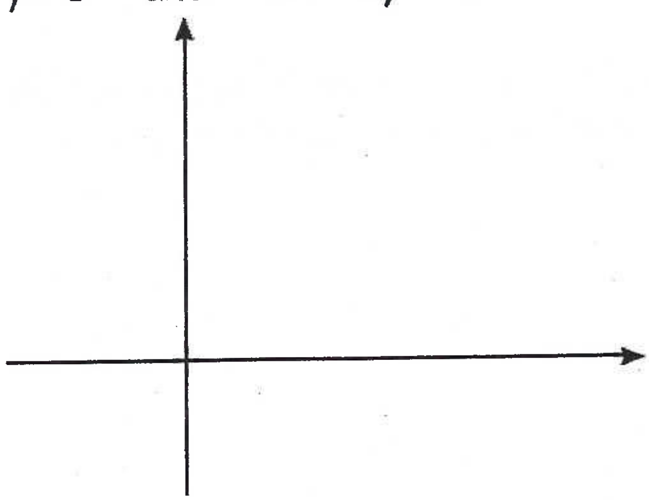
a. Solve for the numbers x and y from the equations $2x - y = 1$ and $3x - 2y = -1$

b. Graph the equations $2x - y = 1$ and $3x - 2y = -1$ and circle the portion of the graph which corresponds to the solution to the above problem on your graph.



c. Graph the solution to the linear inequalities

$2x - y > 1$ and $3x - 2y < -1$



Assessment for Common Core Mathematics Standards Grade 8

10. a. Simplify

1. $3x^2 \; x^4 \; x^5$
2. $\frac{4x^3}{2x}$
3. $6x^2 + 9x^2$

b. Let $P = 2x^2 + 3x - 1$ and $Q = -3x^2 + 4x - 1$

1. Calculate P + Q and collect like terms.
2. Calculate P - Q and collect like terms.

c. Calculate the product $(x^2 - 1)(2x^2 - x - 3)$ and collect like terms.

d. The area of a rectangle is 16. The length of the rectangle is $\frac{x^5}{x + 1}$ and the width is $\frac{x + 1}{x^3}$. What is x?

Assessment for Common Core Mathematics Standards Grade 8

11. Factor the following expressions:

a. $x^2 + 5x + 4$

b. $x^3 + 6x^2 + 9x$

c. $(a + b)x + (a + b)y$

d. $3x^2 + 7x + 2$

e. $p^2 - q^2$

f. $199^2 - 99^2$ (calculate by performing only one multiplication)

12. Reduce to the lowest terms: $\dfrac{x^5 - x^3}{x^2 - 3x + 2}$

Assessment for Common Core Mathematics Standards Grade 8

13. Express each of the following as a quotient of two polynomials reduced to lowest terms

a. $\dfrac{3}{x+1} - \dfrac{4}{x-2}$

b. $\dfrac{x}{2x-1} + \dfrac{x-1}{2x+1} + \dfrac{2x}{4x^2-1}$

c. $\dfrac{a^2-4}{a^3+a} \times \dfrac{4a}{a-2}$

d. $\dfrac{t^2+2t+1}{t+2} \div \dfrac{t+1}{t^2+5t+6}$

14. a. Solve by factoring: $2x^2 - x - 15 = 0$

14.

[CONTINUED]

b. 1. Complete the square of the polynomial $x^2 + 6x + 5$ by finding numbers h and k such that $x^2 + 6x + 5 = (x + h)^2 + k$.

$h =$ _______

$k =$ _______

2. Solve for x if $2(x - 3)^2 - 5 = 0$

15.

a. What percent of $225 is $180?

b. The Smith family is traveling to a vacation destination in two cars. Mrs. Smith leaves home at noon with the children, traveling 40 miles per hour. Mr. Smith leaves 1 hour later and travels at 55 miles per hour.

At what time does Mr. Smith overtake Mrs. Smith?

15.

[CONTINUED]

c. A chemist has one solution of hydrochloric acid and water that is 25% acid and a second that is 75% acid. How many liters of each should be mixed together to get 250 liters of a solution that is 40% acid?

d. Molly can deliver the papers on her route in 2 hours. Tom can deliver the same route in 3 hours. How long would it take them to deliver the papers if they worked together?

16.

17.

18.

a. Which value of x would cause the relation below NOT to be a function?

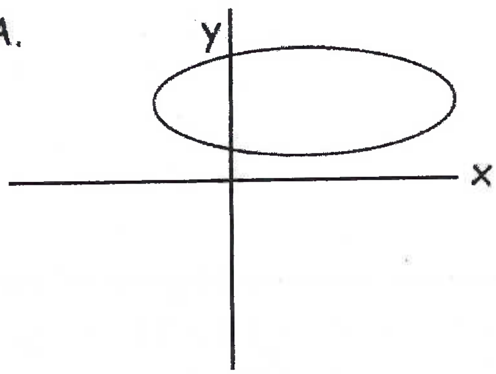
$$\{(1, 3), (x, 7), (6, 8)\}$$

A. 1
B. 3
C. 7
D. 8

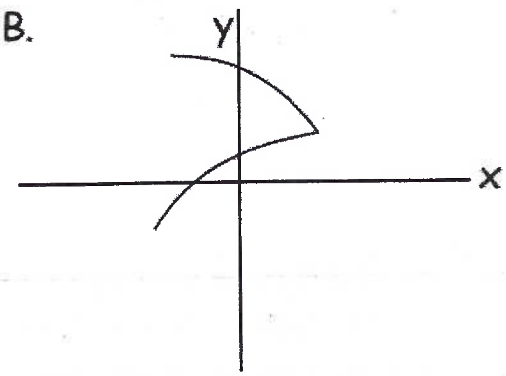
[CONTINUED]

b. Which of the following graphs of relations is also the graph of a function?

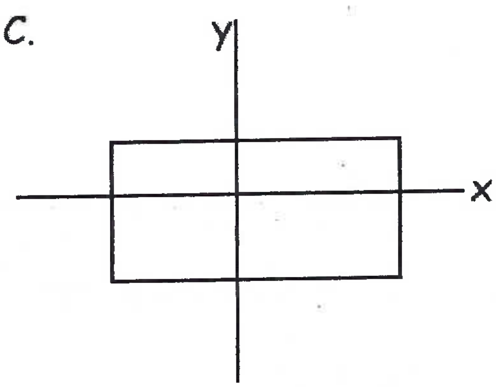
A.



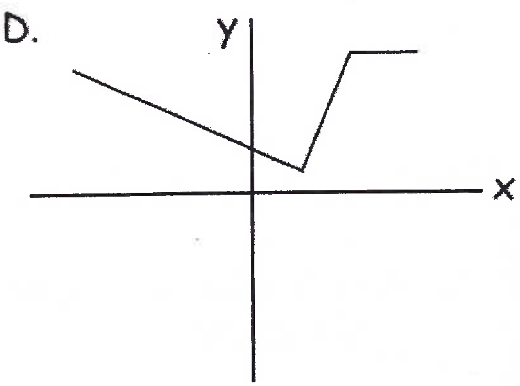
B.



C.



D.



c. Determine the range and the domain of the relation
$\{ (x, y) : x^2 + y^2 = 1 \}$

Domain = ____________

Range = ____________

16.

17.

18.

[CONTINUED]

d. Find the natural domain of the function $f(x) = x^2 / \sqrt{1 + x}$

e. Find the range of the function $g(x) = 5x^2 + 13$

f. Find the range of the relation
$\{(1, 2), (1, 4), (3, 4), (5, 6), (7, 8)\}$

Assessment for Common Core Mathematics Standards Grade 8

19.

a. Given a quadratic equation of the form: $ax^2 + bx + c = 0,\ a \neq 0$
What is the formula for finding the solutions to the equation?

b. The equations below are part of a derivation of the quadratic formula by completing the square:

$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Which of the following is the best next step for the derivation of the quadratic formula?

A. $ax^2 + bx = -c$

B. $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$

C. $\left(x^2 + \frac{b}{a}x\right)^2 = \left(\frac{c}{a}\right)$

D. $\sqrt{x^2 + \frac{b}{a}x} = \sqrt{-\frac{c}{a}}$

20. Find all values of x which satisfy the equation $4x^2 - 4x - 1 = 0$

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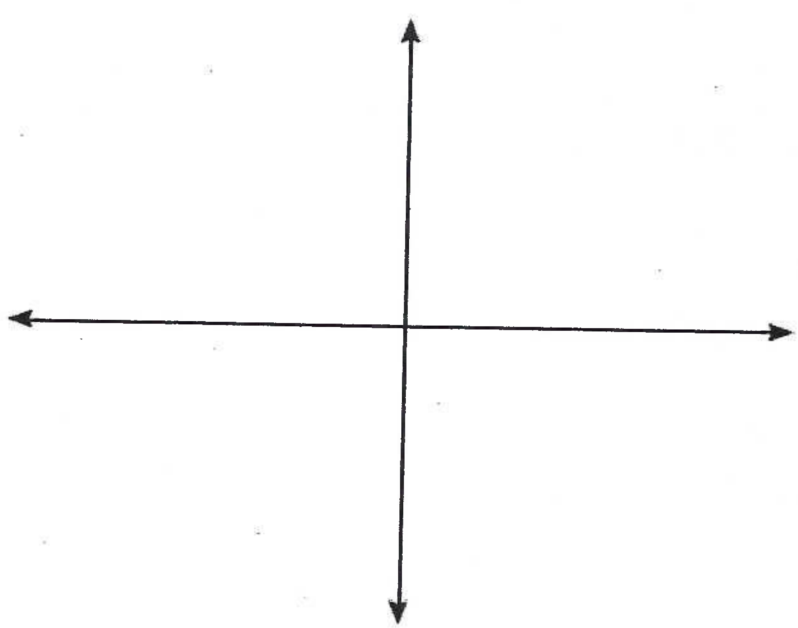
21. You may assume that the following equation is correct for all values of x:

$$-3x^2 + 12x - \frac{21}{2} = -3(x-2)^2 + \frac{3}{2}$$

a. For which values of x, if any, does the graph of the equation $y = -3x^2 + 12x - \frac{21}{2}$ cross the x axis?

b. Sketch the graph of the equation $y = -3x^2 + 12x - \frac{21}{2}$



22. Use the quadratic formula or the method of factoring to determine whether the graphs of the following functions intersect the x axis in zero, one, or two points. (Do not graph the functions.)

a. $y = x^2 + x + 1$

b. $y = 4x^2 + 12x + 5$

c. $y = 9x^2 - 12x + 4$

23.

a. If an object is thrown vertically with an initial velocity of v_o from an initial height of h_o feet, then neglecting air friction its height h(t) in feet above the ground t seconds after the ball was thrown is given by the formula

$$h(t) = -16t^2 + v_o t + h_o$$

If a ball is thrown upward from the top of a 144 foot tower at 96 feet per second, how long will it take for the ball to reach the ground if there is no air friction and the path of the ball is unimpeded?

b. The boiling point of water depends on air pressure and air pressure decreases with altitude. Suppose that the height H above the ground in meters can be deduced from the temperature T at which water boils in degress Celsius by the following formula:

$$H = 1000\,(100 - T) + 580\,(100 - T)^2$$

1. If water on the top of a mountain boils at 99.5 degrees Celsius, how high is the mountain?

2. What is the approximate boiling point of water at sea-level (H=0 meters) according to this equation? Round your answers to the nearest 10 degrees.

Assessment for Common Core Mathematics Standards Grade 8

24

a. Verify to your own satisfaction, by direct calculation, the correctness of the following equations (do not submit your calculations on this exam):

$$3 = \frac{3}{2}(3^1 - 1)$$

$$3 + 3^2 = \frac{3}{2}(3^2 - 1)$$

$$3 + 3^2 + 3^3 = \frac{3}{2}(3^3 - 1)$$

$$3 + 3^2 + 3^3 + 3^4 = \frac{3}{2}(3^4 - 1)$$

1. Using inductive reasoning, propose a formula that gives the sum for $3 + 3^2 + 3^3 + \ldots + 3^n$ for any counting number n.

2. Does the sequence of formulas above prove that your answer to part 1 is correct? Explain your answer.

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24

[CONTINUED]

b. Consider the following mathematical statement:

If y is a positive integer, then $1 + 1141y^2$ is not a perfect square.

1. Write the hypothesis of this statement.

2. Write the conclusion of this statement.

3. Use whole number arithmetic to prove that the conclusion is correct when y = 1.

4. It has been shown by mathematicians that the conclusion is correct for each positive integer y up to and including 30,693,385,322,765,657,197,397,207. However, if this number is increased by 1 so that

$$y = 30{,}693{,}385{,}322{,}765{,}657{,}197{,}397{,}208$$

then the positive square root of $1 + 1141y^2$ is

$$1{,}036{,}782{,}394{,}157{,}223{,}963{,}237{,}125{,}215$$

Is the statement, "If y is a positive integer, then $1 + 1141y^2$ is not a perfect square" correct? Explain your answer.

[CONTINUED]

25

a. Prove, using basic properties of algebra, or disprove by finding a counterexample, each of the following statements:

1. The set of even numbers is closed under addition.

2. The sum of any two odd numbers is even.

3. For any positive real number x, $\sqrt{x} \leq x$

b. Find all possible pairs of numbers a and b which satisfy the equation $a^2 + b^2 = (a + b)^2$. Explain your reasoning.

c. Identify the step below in which a fallacy occurs:

Step 1: Let $a = b = 1$
Step 2: $a^2 = ab$
Step 3: $a^2 - b^2 = ab - b^2$
Step 4: $(a - b)(a + b) = b(a - b)$
Step 5: $a + b = b$
Step 6: $2 = 1$

Answer: Step ______
Explain why the step you have chosen as the fallacy is incorrect.

25

[CONTINUED]

d. Is the following equation true for some values of x, no values of x or all values of x?

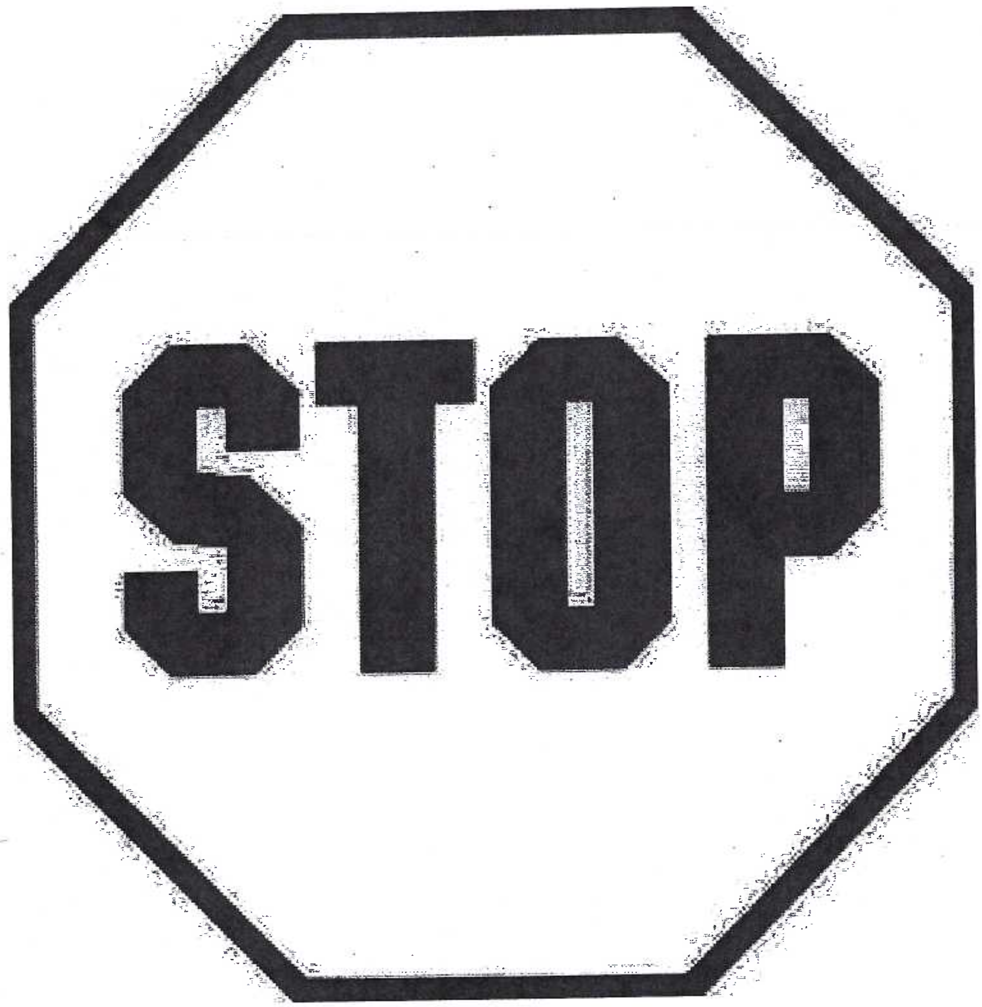
$$16\left(x - \frac{1}{4}\right)^2 - 1 = 16x^2 - 8x$$

Assessment for Common Core Mathematics Standards Grade 8

End of Assessment

S1.2

GRADE EIGHT



Answer Key

Algebra I: Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem-solving situations.

1.0: Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable.

a. Fill in the blanks below with a single appropriate letter to identify each set of numbers with the properties or descriptions of the elements which characterize that set:

The set of:

E	Even Numbers	B	Integers
D	Rational Numbers	F	Odd Numbers
G	Irrational Numbers	J	Natural Numbers
C	Real Numbers	H	Whole Numbers

A. any number equal to a terminating decimal expression

B. {..., -3, -2, -1, 0, 1, 2, 3, ...}

C. any number which is rational or irrational

D. any number of the form $\frac{p}{q}$ where p and q are integers and q is not zero

E. any integer of the form 2k, where k is an integer

F. any integer of the form 2k + 1, where k is an integer

G. any number equal to an infinite decimal expression with no repeating block of digits

H. {0, 1, 2, 3, ...}

I. any number which can be expressed as a ratio

J. {1, 2, 3, ...}

[CONTINUED ON NEXT PAGE]

Answer Key

1.0: Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable.

[CONTINUED]

b.

Which of the following sets of numbers are not closed under addition?

__ The set of real numbers

__ The set of rational numbers

X The set of irrational numbers

__ The set of positive integers

Answer Key

2.0: Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

a. Which number below is the same as $-\frac{2}{3}-(-(-\frac{3}{5}))$?

A. $\frac{4}{15}$ B. $-\frac{4}{15}$ C. $-\frac{1}{2}$ D. $-\frac{19}{15}$

$$-\frac{2}{3}-(-(-\frac{3}{5})) = -\frac{2}{3} + -\frac{3}{5}$$
$$= -(\frac{2}{3} + \frac{3}{5})$$
$$= -(\frac{10}{15} + \frac{9}{15})$$
$$= -\frac{19}{15}$$

b. What number z satisfies the equation $\frac{2}{3}z = 1$?

$$\frac{2}{3}z = 1$$
$$\frac{3}{2}(\frac{2}{3}z) = \frac{3}{2} \cdot 1$$
$$(\frac{3}{2} \cdot \frac{2}{3})z = \frac{3}{2}$$
$$z = \frac{3}{2}$$

Answer Key

2.0: Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

[CONTINUED]

c. Calculate and simplify the expression $\sqrt[3]{2\sqrt{16}}$

$$\sqrt[3]{2\sqrt{16}} = \sqrt[3]{2 \cdot 4} = \sqrt[3]{8} = 2$$

d. If x = 4, what is $x^{-3/2}$ (x^{100} / x^{99})?

$$x^{-3/2}\,(x^{100} / x^{99}) = x^{-3/2} \cdot x$$

$$= x^{-3/2+1}$$

$$= x^{-1/2}$$

When x = 4, $x^{-1/2} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

Answer Key

2.0: Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

[CONTINUED]

e. Write $\dfrac{(x^{3/2})^{4/3}\sqrt[4]{x}}{x}$ as x raised to a power

$$\frac{(x^{3/2})^{4/3}\sqrt[4]{x}}{x} = x^{3/2 \cdot 4/3} \cdot \frac{x^{1/4}}{x}$$

$$= x^2 \cdot x^{1/4 - 1}$$

$$= x^2 \cdot x^{-3/4}$$

$$= x^{2-3/4}$$

$$= x^{5/4}$$

Answer Key

3.0: Students solve equations and inequalities involving absolute values.

a $|2x - 1| = 5$ Find all values of x which make this equation true

If $|2x - 1| = 5$, then $2x - 1 = 5$ or $2x - 1 = -5$

Therefore $x = 3$ or $x = -2$

b. $3|2 - 5x| + 1 < 10$ Find all values of x which make this inequality true

$3|2 - 5x| + 1 < 10$

$3|2 - 5x| < 9$

$|2 - 5x| < 3$

$-3 < 2 - 5x < 3$

$-5 < -5x < 1$

$1 > x > -\frac{1}{5}$

Solution: All numbers greater than $-\frac{1}{5}$ and less than 1

c. $4|x - 1| = 16$ Find all values of x which make this equation true

$4|x - 1| = 16$

$|x - 1| = 4$

$x - 1 = 4$ or $x - 1 = -4$

$x = 5$ or $x = -3$

3.0: Students solve equations and inequalities involving absolute values.

[CONTINUED]

d. $|x-2| > 4$ Find all values for x which make this inequality true

Either $x - 2 > 4$ or $x - 2 < -4$

Either $x > 6$ or $x < -2$

Solution: All numbers less than -2 or greater than 6

4.0: Students simplify expressions before solving linear equations and inequalities in one variable, such as 3(2x-5) + 4 (x-2) = 12.

a. Simplify

1. $3(2x - 5) + 4(x - 2)$

$$\begin{aligned} 3(2x - 5) + 4(x - 2) &= 6x - 15 + 4x - 8 \\ &= 6x + 4x - 15 - 8 \\ &= 10x - 23 \end{aligned}$$

2. $\frac{1}{2}(6x + 4) - \frac{1}{3}(3 - 6x)$

$$\begin{aligned} \frac{1}{2}(6x + 4) - \frac{1}{3}(3 - 6x) &= 3x + 2 - 1 + 2x \\ &= 3x + 2x + 2 - 1 \\ &= 5x + 1 \end{aligned}$$

Answer Key

4.0: Students simplify expressions before solving linear equations and inequalities in one variable, such as 3(2x-5) + 4 (x-2) = 12.

[CONTINUED]

b. Solve for x

1. $8(x + 1) + 3(2x - 2) = 44$

$$8(x + 1) + 3(2x - 2) = 44$$

$$8x + 8 + 6x - 6 = 44$$

$$8x + 6x + 8 - 6 = 44$$

$$14x + 2 = 44$$

$$14x = 42$$

$$x = \frac{42}{14}$$

$$x = 3$$

2. $\frac{1}{3}(12x - 9) - 2 (x - 5) \geq 17$

$$\frac{1}{3}(12x - 9) - 2 (x - 5) \geq 17$$

$$4x - 3 - 2x + 10 \geq 17$$

$$2x + 7 \geq 17$$

$$2x \geq 10$$

$$x \geq 5$$

Answer Key

5.0: Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

a. Justify each step below for the solution for x from the equation

$$\frac{2}{3}(x+3)+4(x-8)=2$$

Use the following list:

A. **Commutative Property of Addition**
B. **Associative Property of Addition**
C. **Commutative Property of Multiplication**
D. **Associative Property of Multiplication**
E. **Distributive Property**
F. **adding the same quantity to both sides of an equation preserves equality**
G. **multiplying both sides of an equation by the same number preserves equality**
H. **0 is the additive identity**
I. **1 is the multiplicative identity**

To the right of each equation below (and on the following pages) where there is an empty space, write one of the letters 'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', or 'I' to justify how that equation follows from the one above it. For example, the second equation below is justified by 'G' and the third one by 'E'.

[CONTINUES ON NEXT PAGE]

Answer Key

5.0: Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

[CONTINUED]

Step	**Justification**
$\frac{2}{3}(x+3)+4(x-8)=2$	The given equation
$\frac{3}{2}\left[\frac{2}{3}(x+3)+4(x-8)\right]=\left(\frac{3}{2}\right)2$	G
$(x+3)+6(x-8)=3$	E
$(x+3)+(6x-48)=3$	E
$[(x+3)+6x]-48=3$	B
$[x+(3+6x)]-48=3$	B
$[x+(6x+3)]-48=3$	A
$[(x+6x)+3]-48=3$	B
$[(1+6)x+3]-48=3$	E
$(7x+3)-48=3$	$1+6=7$
$7x+(3-48)=3$	B

Answer Key

5.0: Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

[CONTINUED]

Step	**Justification**
$7x - 45 = 3$	$3 - 48 = -45$
$(7x - 45) + 45 = 3 + 45$	F
$7x + (-45 + 45) = 48$	B
$7x + 0 = 48$	$-45 + 45 = 0$
$7x = 48$	H
$\frac{1}{7}(7x) = \frac{48}{7}$	G
$(\frac{1}{7} 7)x = \frac{48}{7}$	D
$1x = \frac{48}{7}$	$\frac{1}{7} \cdot 7 = 1$
$x = \frac{48}{7}$	I

[CONTINUED ON NEXT PAGE]

Answer Key

5.0: Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

b. The sum of three integers is 66. The second is 2 more than the first, and the third is 4 more than twice the first.

What are the integers?

Let x be the first number. The the second number is x + 2 and the third number is 2x + 4.

$x + (x + 2) + (2x + 4) = 66$
$4x + 6 = 66$
$4x = 60$
$x = 15$
$x + 2 = 17$
$2x + 4 = 34$
The three numbers are 15, 17, and 34

c. During an illness, a patient's body temperature T satisfied the inequality $|T - 98.6| \le 2$. Find the lowest temperature the patient could have had during the illness.

$|T - 98.6| \le 2$
$-2 \le T - 98.6 \le 2$
$-2 + 98.6 \le T \le 2 + 98.6$
$96.6 \le T \le 100.6$

The lowest temperature the patient could have had is 96.6

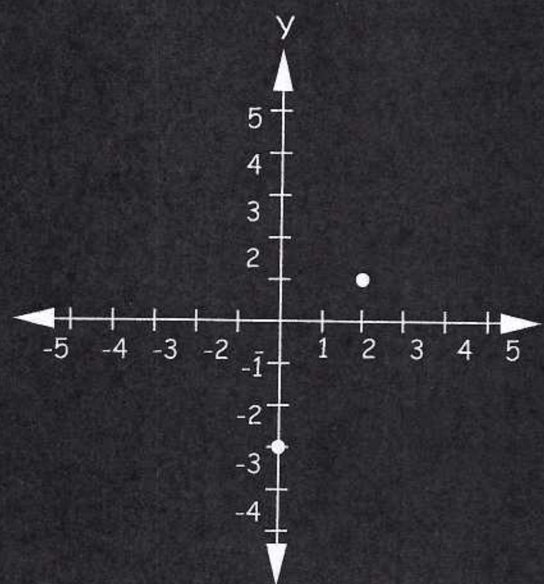
Answer Key

6.0: Students graph a linear equation and compute the *x*- and *y*-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear equality (e.g., they sketch the region defined by $2x + 6y < 4$).

a. Graph the equation: $2x - y = 3$

$$2x - y = 3$$
$$y = 2x - 3$$

x	y
0	-3
2	1



Two points on the line are (0,-3) and (2,1). Note: Two points determine a line. Calculating the coordinates of any two points on the line determines the same graph.

b. What is the x intercept?

In the equation $2x - y = 3$ (or $y = 2x - 3$), substitute $y = 0$.

$$2x - 0 = 3$$
$$2x = 3$$
$$x = \frac{3}{2}$$

[CONTINUED ON NEXT PAGE]

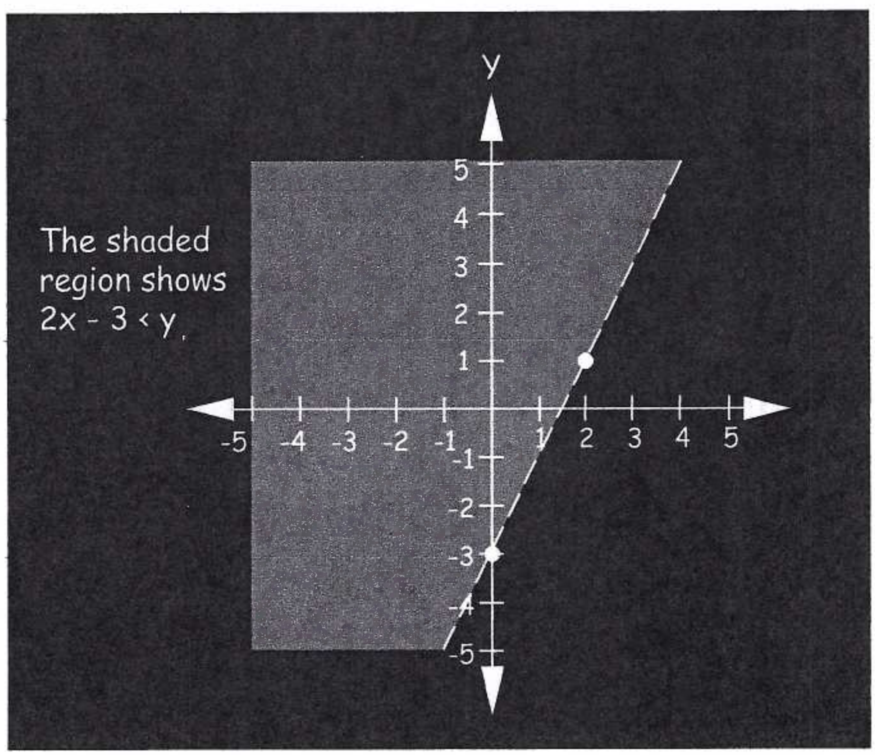
6.0: Students graph a linear equation and compute the *x*- and *y*-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear equality (e.g., they sketch the region defined by $2x + 6y < 4$).

[CONTINUED]

c. What is the y intercept?

In the equation $2x - y = 3$, substitute $x = 0$ to get $y = -3$. Alternatively, since b is the y intercept for $y = mx + b$, it follows that -3 is the y intercept for $y = 2x - 3$.

d. On your graph, mark the region showing $2x - 3 < y$



Answer Key

7.0: Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.

a. Write an equation involving only numbers that shows that the point $(1\frac{1}{2}, 2)$ lies on the graph of the equation $2y = 6x - 5$.

$$2 \cdot 2 = 6(1\tfrac{1}{2}) - 5 \quad \text{or} \quad 2 \cdot 2 = 6(\tfrac{3}{2}) - 5$$

b. A line has a slope of $\frac{1}{2}$ and passes through the point (5, 8).
What is the equation for the line?

The equation of the line must be of the form $y = mx + b$. The slope $m = \frac{1}{2}$ is given. Therefore, $y = \frac{1}{2}x + b$. To find the y intercept b, substitute the coordinates of the point (5,8) for x and y in the equation $y = \frac{1}{2}x + b$. This gives:

$$8 = \frac{1}{2} \cdot 5 + b$$
$$b = 8 - \frac{5}{2}$$
$$b = \frac{16 - 5}{2}$$
$$b = \frac{11}{2}$$
$$\text{Therefore } y = \frac{1}{2}x + \frac{11}{2}$$

This result may also be obtained by using the point-slope formula for a nonvertical line: $y - y_0 = m(x - x_0)$ and substituting $m = \frac{1}{2}$, $x_0 = 5$, and $y_0 = 8$.

Answer Key

8.0: Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

a. A line is parallel to the line for the equation:
$\frac{1}{2}y = \frac{1}{2}x - 9$. What is the slope of the parallel line?

$\frac{1}{2}y = \frac{1}{2}x - 9$ may be rewritten as $y = x - 18$, which has slope 1. Any line parallel to this one must have the same slope, 1.

b. What is the slope of a line perpendicular to the line for the equation $3y = 7 - 6x$?

$3y = 7 - 6x$ may be rewritten as $y = -2x + \frac{7}{3}$. The slope m of any line perpendicular to this one must satisfy $m(-2) = -1$. Therefore $m = \frac{1}{2}$.

c. What is the equation of a line passing through the point (7, 4) and perpendicular to the line having the equation $3x - 4y - 12 = 0$?

The equation $3x - 4y - 12 = 0$ may be rewritten as $y = \frac{3}{4}x - 3$. The slope of this line is $\frac{3}{4}$. The slope m of any line perpendicular to this one must satisfy $m(\frac{3}{4}) = -1$. Therefore $m = -\frac{4}{3}$. So the equation of any perpendicular line must be of the form $y = -\frac{4}{3}x + b$. Since the graph of the line contains the point (7,4), it is also true that

$$4 = -\frac{4}{3}(7) + b$$

$$4 = -\frac{28}{3} + b$$

$$b = \frac{12}{3} + \frac{28}{3} = \frac{40}{3}$$

So the answer is $y = -\frac{4}{3}x + \frac{40}{3}$. This answer may also be obtained by using the point-slope formula $y - y_o = m(x - x_o)$ with $m = -\frac{4}{3}$, $x_o = 7$, and $y_o = 4$.

Answer Key

9.0: Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

a. Solve for the numbers x and y from the equations $2x - y = 1$ and $3x - 2y = -1$

$2x - y = 1$ $3x - 2y = -1$	Multiply the first equation by -2 and then add it to the second equation
$-4x + 2y = -2$ $3x - 2y = -1$	
$-x + 0y = -3$ $x = 3$	Solve for x
$2 \cdot 3 - y = 1$	Substitute $x = 3$ into the first equation and solve for y
$6 - y = 1$ $y = 5$	
$x = 3, y = 5$	Solution

There are other ways to solve this problem. One can use one of the equations to solve for one of the variables in terms of the other and substitute that expression into the other equation. For example, from $2x - y = 1$, solving for y gives $y = 2x - 1$. Substituting this expression for y in the second equation gives

$$3x - 2(2x - 1) = -1$$
$$3x - 4x + 2 = -1$$
$$-x + 2 = -1$$
$$-x = -3$$
$$x = 3$$

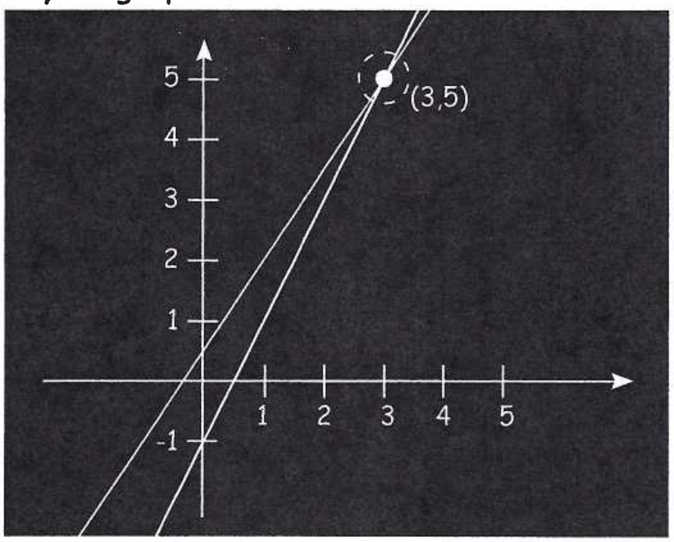
Now substituting $x = 3$ into either equation yields $y = 5$. So the solution again is $x = 3$ and $y = 5$.

Answer Key

9.0: Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

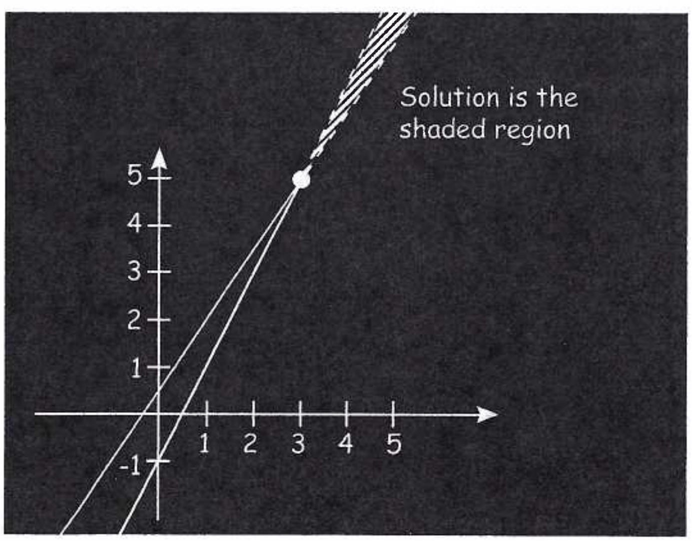
[CONTINUED]

b. Graph the equations $2x - y = 1$ and $3x - 2y = -1$ and circle the portion of the graph which corresponds to the solution to the above problem on your graph.



c. Graph the solution to the linear inequalities

$2x - y > 1$ and $3x - 2y < -1$



Answer Key

10.0: Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.

a. Simplify

1. $3x^2\ x^4\ x^5$

$3x^2 x^4 x^5$
$= 3x^{2+4+5}$
$= 3x^{11}$

2. $\frac{4x^3}{2x}$

$\frac{4x^3}{2x} = \frac{4}{2} \cdot \frac{x^3}{x}$
$= 2x^{3-1}$
$= 2x^2$

3. $6x^2 + 9x^2$

$6x^2 + 9x^2 = (6 + 9)x^2$
$= 15x^2$

b. Let $P = 2x^2 + 3x - 1$ and $Q = -3x^2 + 4x - 1$

1. Calculate P + Q and collect like terms.

$P + Q = 2x^2 + 3x - 1 + (-3x^2 + 4x - 1)$
$= (2x^2 - 3x^2) + (3x + 4x) - 1 - 1$
$= -x^2 + 7x - 2$

2. Calculate P - Q and collect like terms.

$P - Q = 2x^2 + 3x - 1 - (-3x^2 + 4x -1)$
$= 2x^2 + 3x - 1 + 3x^2 - 4x + 1$
$= (2x^2 + 3x^2) + (3x - 4x) - 1 + 1$
$= 5x^2 - x$

c. Calculate the product $(x^2 - 1)(2x^2 - x - 3)$ and collect like terms.

$(x^2 - 1)(2x^2 - x - 3)$
$= (x^2 - 1)2x^2 + (x^2 - 1)(-x) + (x^2 - 1)(-3)$
$= 2x^4 - 2x^2 - x^3 + x - 3x^2 + 3$
$= 2x^4 - x^3 - 5x^2 + x + 3$

Answer Key

10.0: Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.

[CONTINUED]

d. The area of a rectangle is 16. The length of the rectangle is $\frac{x^5}{x+1}$ and the width is $\frac{x+1}{x^3}$. What is x?

A = length times width

$$16 = \frac{x^5}{x+1} \cdot \frac{x+1}{x^3}$$

$$16 = \frac{x^5}{x^3}$$

$$16 = x^2$$

$$x = 4$$

Answer Key

11.0: Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Factor the following expressions:

a. $x^2 + 5x + 4$

$x^2 + 5x + 4 = (x + 4)(x + 1)$

b. $x^3 + 6x^2 + 9x$

$x^3 + 6x^2 + 9x = x(x^2 + 6x + 9) = x(x + 3)^2$

c. $(a + b)x + (a + b)y$

$(a + b)x + (a + b)y = (a + b)(x + y)$

d. $3x^2 + 7x + 2$

$3x^2 + 7x + 2 = (3x + 1)(x + 2)$

e. $p^2 - q^2$

$p^2 - q^2 = (p - q)(p + q)$

f. $199^2 - 99^2$ (calculate by performing only one multiplication)

$199^2 - 99^2 = (199 - 99)(199 + 99) = 100 \cdot 298 = 29{,}800$

Answer Key

12.0: Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.

Reduce to the lowest terms: $\frac{x^5 - x^3}{x^2 - 3x + 2}$

$$\frac{x^5 - x^3}{x^2 - 3x + 2}$$

$$= \frac{x^3(x^2 - 1)}{(x - 2)(x - 1)}$$

$$= \frac{x^3(x + 1)(x - 1)}{(x - 2)(x - 1)}$$

$$= \frac{x^3(x + 1)}{x - 2}$$

Answer Key

13.0: Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

Express each of the following as a quotient of two polynomials reduced to lowest terms

a. $\frac{3}{x+1} - \frac{4}{x-2}$

$$\frac{3}{x+1} - \frac{4}{x-2} = \frac{3(x-2)}{(x+1)(x-2)} - \frac{4(x+1)}{(x+1)(x-2)} =$$

$$\frac{3x-6-4x-4}{(x+1)(x-2)} = \frac{-(x+10)}{(x+1)(x-2)} \text{ or } \frac{-x-10}{x^2-x-2} \text{ or } -\frac{x+10}{x^2-x-2} \text{ etc.}$$

b. $\frac{x}{2x-1} + \frac{x-1}{2x+1} + \frac{2x}{4x^2-1}$

$$\frac{x}{2x-1} + \frac{x-1}{2x+1} + \frac{2x}{4x^2-1} = \frac{x(2x+1)}{(2x-1)(2x+1)} + \frac{(x-1)(2x-1)}{(2x+1)(2x-1)} + \frac{2x}{4x^2-1}$$

$$= \frac{2x^2+x}{4x^2-1} + \frac{2x^2-3x+1}{4x^2-1} + \frac{2x}{4x^2-1}$$

$$= \frac{(2x^2+x)+(2x^2-3x+1)+2x}{4x^2-1} = \frac{4x^2+1}{4x^2-1}$$

Answer Key

13.0: Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

[CONTINUED]

Express each of the following as a quotient of two polynomials reduced to lowest terms

c. $\frac{a^2 - 4}{a^3 + a} \times \frac{4a}{a - 2}$

$$\frac{a^2 - 4}{a^3 + a} \cdot \frac{4a}{a - 2} = \frac{(a - 2)(a + 2)}{a(a^2 + 1)} \cdot \frac{4a}{a - 2} = \frac{4(a + 2)}{a^2 + 1}$$

d. $\frac{t^2 + 2t + 1}{t + 2} \div \frac{t + 1}{t^2 + 5t + 6}$

$$\frac{t^2 + 2t + 1}{t + 2} \div \frac{t + 1}{t^2 + 5t + 6} = \frac{t^2 + 2t + 1}{t + 2} \cdot \frac{t^2 + 5t + 6}{t + 1} =$$

$$\frac{(t + 1)^2}{t + 2} \cdot \frac{(t + 2)(t + 3)}{t + 1} = (t + 1)(t + 3) = t^2 + 4t + 3$$

provided $t \neq -1, -2$, or -3

14.0: Students solve a quadratic equation by factoring or completing the square.

a. Solve by factoring: $2x^2 - x - 15 = 0$

$2x^2 - x - 15 = 0$

$(2x + 5)(x - 3) = 0$

Either $2x + 5 = 0$ or $x - 3 = 0$

Either $x = -\frac{5}{2}$ or $x = 3$

b. 1. Complete the square of the polynomial $x^2 + 6x + 5$ by finding numbers h and k such that $x^2 + 6x + 5 = (x + h)^2 + k$.

$h = 3$
$k = -4$

$$\begin{aligned} x^2 + 6x + 5 &= x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 5 \\ &= (x^2 + 6x + 9) - 9 + 5 \\ &= (x + 3)^2 - 4 \end{aligned}$$

2. Solve for x if $2(x - 3)^2 - 5 = 0$

$2(x - 3)^2 - 5 = 0$

$2(x - 3)^2 = 5$

$(x - 3)^2 = \frac{5}{2}$

$x - 3 = \pm\sqrt{\frac{5}{2}}$

$x = 3 \pm \sqrt{\frac{5}{2}}$

The two roots are $3 + \sqrt{\frac{5}{2}}$ and $3 - \sqrt{\frac{5}{2}}$

Answer Key

15.0: Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

a. What percent of \$225 is \$180?

$\frac{180}{225} = \frac{n}{100}$ $\qquad \frac{4}{5} = \frac{n}{100}$

$5n = 4 \cdot 100$ $\qquad n = \frac{400}{5}$

$n = 80$ $\qquad$ \$180 is 80% of \$225

b. The Smith family is traveling to a vacation destination in two cars. Mrs. Smith leaves home at noon with the children, traveling 40 miles per hour. Mr. Smith leaves 1 hour later and travels at 55 miles per hour.

At what time does Mr. Smith overtake Mrs. Smith?

A car travelling for t hours at a rate of r miles per hour travels a distance of d miles where $d = rt$.

Let d_1 = distance traveled by Mrs Smith t hours past 1:00 p.m.
Let d_2 = distance traveled by Mr Smith t hours past 1:00 p.m.

Then $d_1 = 40(t + 1)$ and $d_2 = 55t$

To find the number of hours past 1:00 p.m. at which time they meet, set $d_1 = d_2$

$d_1 = d_2$ $\qquad 40(t + 1) = 55t$

$40t + 40 = 55t$ $\qquad 15t = 40$

$t = \frac{40}{15}$ hours $\qquad t = 2\frac{2}{3}$ hours

t = 2 hours and 40 minutes

Mrs Smith and Mr Smith will meet at 3:40 p.m.

15.0: Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

[CONTINUED]

c. A chemist has one solution of hydrochloric acid and water that is 25% acid and a second that is 75% acid. How many liters of each should be mixed together to get 250 liters of a solution that is 40% acid?

Let x = the no. of liters needed of the 25% acid solution.
Let y = the no. of liters needed of the 75% acid solution.

The mixture must satisfy two equations: $x + y = 250$, and $(.25)x + (.75)y = (.40)250$. The second equation may be rewritten as $\frac{1}{4}x + \frac{3}{4}y = \frac{2}{5}(250)$ or $\frac{1}{4}x + \frac{3}{4}y = 100$. Multiplying both sides by 4 gives $x + 3y = 400$. Subtracting the first equation from this last one gives:

$$\begin{array}{r} x + 3y = 400 \\ \underline{x + y \;\; = 250} \\ 2y \;\;\;\; = 150 \end{array}$$

So $y = 75$. Substituting $y = 75$ into either equation gives $x = 175$. The answer is that 175 liters of the 25% acid solution must be mixed with 75 liters of 75% acid solution to produce 250 liters of a 40% acid solution.

d. Molly can deliver the papers on her route in 2 hours. Tom can deliver the same route in 3 hours. How long would it take them to deliver the papers if they worked together?

If it takes Molly 2 hours to complete the job, she does $\frac{1}{2}t$ jobs in t hours. If it takes Tom 3 hours to complete the job, he can do $\frac{1}{3}t$ jobs in t hours. To find the number of hours it takes to complete the job with both of them working, solve the equation:

$$\frac{1}{2}t + \frac{1}{3}t = 1$$

$$\left(\frac{1}{2} + \frac{1}{3}\right)t = 1$$

$$\frac{5}{6}t = 1$$

$$t = \frac{6}{5} \text{ or 1 hour and 12 minutes}$$

Answer Key

16.0: Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17.0: Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
18.0: Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

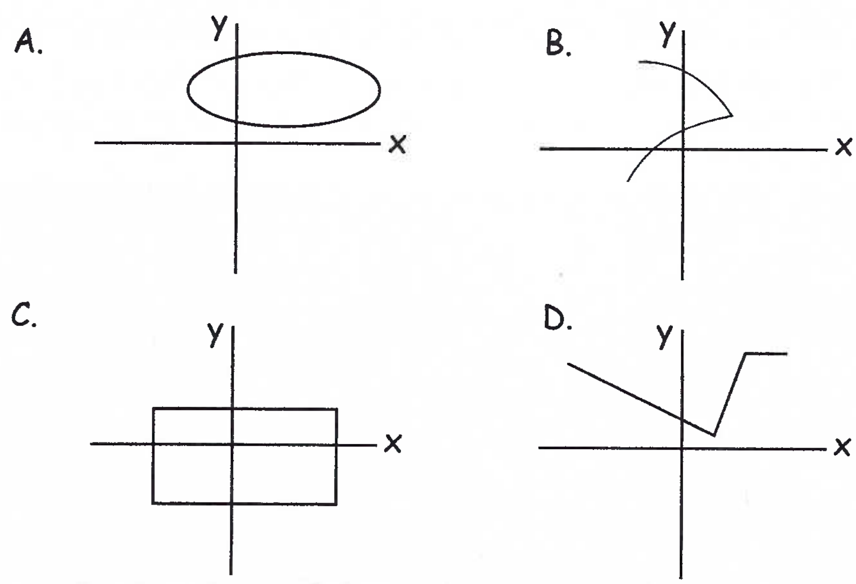
a. Which value of x would cause the relation below NOT to be a function?

$$\{(1, 3), (x, 7), (6, 8)\}$$

A. 1
B. 3
C. 7
D. 8

A function is a relation satisfying the condition that for each element in the domain, there is one and only one element in the range.

b. Which of the following graphs of relations is also the graph of a function?



D. is the only graph satisfying the vertical line test

Answer Key

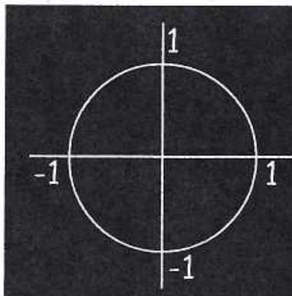
16.0: Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17.0: Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
18.0: Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

[CONTINUED]

c. Determine the range and the domain of the relation
$\{ (x, y) : x^2 + y^2 = 1 \}$

Domain = [-1, 1]

Range = [-1, 1]



The graph of this relation is the unit circle. The x values range from -1 to 1 and the y values range from -1 to 1.

d. Find the natural domain of the function $f(x) = x^2 / \sqrt{1 + x}$

The natural domain is the largest subset of the set of real numbers for which the formula is meaningful. In order to avoid dividing by zero or taking the square root of a negative number, we must require $1 + x > 0$ or $x > -1$. The answer is $\{x : x > -1\}$

Answer Key

16.0: Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17.0: Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
18.0: Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

e. Find the range of the function $g(x) = 5x^2 + 13$

The range of g(x) is the set of all values the function g(x) can take. The range of g(x) is the set of all numbers greater than or equal to 13, i.e. the range is $\{y : y \geq 13\}$. This is because $5x^2 \geq 0$ for any value of x and there is a value of x such that $5x^2$ is equal to any given non-negative number.

f. Find the range of the relation

$\{ (1, 2), (1, 4), (3, 4), (5, 6), (7, 8) \}$

The range of a relation is the set of all second coordinates, in this case, {2, 4, 6, 8}. Notice that 4 is listed only once.

Answer Key

19.0: Students know the quadratic formula and are familiar with its proof by completing the square.

a. Given a quadratic equation of the form: $ax^2 + bx + c = 0, \ a \neq 0$
What is the formula for finding the solutions to the equation?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

b. The equations below are part of a derivation of the quadratic formula by completing the square:

$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Which of the following is the best next step for the derivation of the quadratic formula?

A. $ax^2 + bx = -c$

(B) $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$

C. $\left(x^2 + \frac{b}{a}x\right)^2 = \left(\frac{c}{a}\right)$

D. $\sqrt{x^2 + \frac{b}{a}x} = \sqrt{-\frac{c}{a}}$

The derivation of the quadratic formula by completing the square is standard material in any good Algebra I textbook.

Answer Key

20.0: Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.

Find all values of x which satisfy the equation $4x^2 - 4x - 1 = 0$

$$4x^2 - 4x - 1 = 0$$

$$x = \frac{- -4 \pm \sqrt{(-4)^2 - 4 \cdot 4(-1)}}{2.4}$$

$$x = \frac{4 \pm \sqrt{16 + 16}}{8}$$

$$x = \frac{4 \pm \sqrt{32}}{8}$$

$$x = \frac{4 \pm 4\sqrt{2}}{8}$$

$$x = \frac{1 \pm \sqrt{2}}{2}$$

The roots are $\frac{1 + \sqrt{2}}{2}$ and $\frac{1 - \sqrt{2}}{2}$

Answer Key

21.0: Students graph quadratic functions and know that their roots are the x-intercepts.

You may assume that the following equation is correct for all values of x:

$$-3x^2 + 12x - \frac{21}{2} = -3(x - 2)^2 + \frac{3}{2}$$

a. For which values of x, if any, does the graph of the equation $y = -3x^2 + 12x - \frac{21}{2}$ cross the x axis?

$$-3(x - 2)^2 + \frac{3}{2} = 0$$

$$-3(x - 2)^2 = -\frac{3}{2}$$

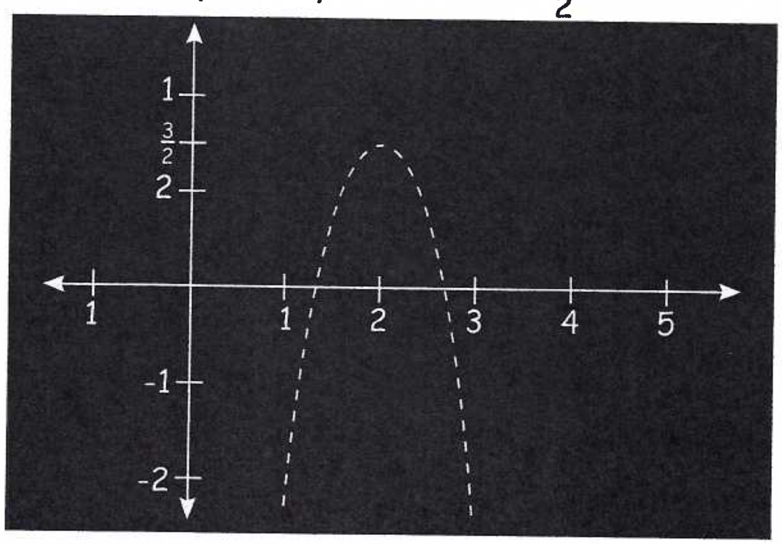
$$(x - 2)^2 = \frac{1}{2}$$

$$x - 2 = \pm\sqrt{\frac{1}{2}}$$

$$x = 2 \pm \sqrt{\frac{1}{2}} = 2 \pm \frac{1}{\sqrt{2}}$$

The graph of $y = -3x^2 + 12x - \frac{21}{2}$ crosses the x-axis at $x = 2 + \frac{1}{\sqrt{2}}$ and $x = 2 - \frac{1}{\sqrt{2}}$

b. Sketch the graph of the equation $y = -3x^2 + 12x - \frac{21}{2}$



Answer Key

22.0: Students use the quadratic formula or factoring techniques or both to determine whether the graph of quadratic function will intersect the x-axis in zero, one or two points.

Use the quadratic formula or the method of factoring to determine whether the graphs of the following functions intersect the x axis in zero, one, or two points. (Do not graph the functions.)

a. $y = x^2 + x + 1$
b. $y = 4x^2 + 12x + 5$
c. $y = 9x^2 - 12x + 4$

Each of these problems may be solved using the discriminant, $D = b^2 - 4ac$, which appears under the radical sign in the quadratic formula. If $D > 0$, the graph has exactly two x-intercepts. If $D = 0$, the graph has exactly one x-intercept. If $D < 0$, the graph does not intersect the x-axis.

a. $D = b^2 - 4ac = 1^2 - 4 \cdot 1 \cdot 1 < 0$

Therefore the graph of $y = x^2 + x + 1$ does not intersect the x-axis (equivalently $x^2 + x + 1 = 0$ has no real solutions).

Answer: 0

b. $D = b^2 - 4ac = 12^2 - 4 \cdot 4 \cdot 5 > 0$. Therefore $y = 4x^2 + 12x + 5$ has two x-intercepts. This may also be seen by factoring:

$$4x^2 + 12x + 5 = (2x + 5)(2x + 1)$$

So the intercepts are $-\frac{5}{2}$ and $-\frac{1}{2}$.

Answer: 2

c. $D = b^2 - 4ac = (-12)^2 - 4 \cdot 9 \cdot 4 = 12^2 - (3 \cdot 4)(3 \cdot 4) = 12^2 - 12^2 = 0$. Therefore $y = 9x^2 - 12x + 4$ has exactly one x-intercept. This may also be seen by factoring:

$$9x^2 - 12x + 4 = (3x - 2)^2$$

Setting this expression equal to zero gives exactly one solution, $x = \frac{3}{2}$.

Answer: 1

Answer Key

23.0: Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.

a. If an object is thrown vertically with an initial velocity of v_0 from an initial height of h_0 feet, then neglecting air friction its height h(t) in feet above the ground t seconds after the ball was thrown is given by the formula

$$h(t) = -16t^2 + v_0 t + h_0$$

If a ball is thrown upward from the top of a 144 foot tower at 96 feet per second, how long will it take for the ball to reach the ground if there is no air friction and the path of the ball is unimpeded?

Let h(t) be the height above the ground at time t measured in seconds. Then

$$h(t) - 16t^2 + 96t + 144$$

In order to find t such that h(t) is zero, set h(t) = 0 and solve for t.

$$-16t^2 + 96t + 144 = 0$$

$$t^2 - 6t - 9 = 0$$

$$t = \frac{6 \pm \sqrt{36 - 4(-9)}}{2}$$

$$t = \frac{6 \pm \sqrt{36 \cdot 2}}{2}$$

$$t = \frac{6 \pm 6\sqrt{2}}{2} = 3 \pm 3\sqrt{2}$$

Since the object was thrown at t = 0 and time moves forward, the correct solution is t= $3 + 3\sqrt{2}$ seconds.

23.0: Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.

[CONTINUED]

b. The boiling point of water depends on air pressure and air pressure decreases with altitude. Suppose that the height H above the ground in meters can be deduced from the temperature T at which water boils in degress Celsius by the following formula:

$$H = 1000(100 - T) + 580(100 - T)^2$$

1. If water on the top of a mountain boils at 99.5 degrees Celsius, how high is the mountain?

$H = 1{,}000(100 - 99.5) + 580(100 - 99.5)^2 = 1{,}000(\frac{1}{2}) + 580(\frac{1}{4})$

$H = 500 + 145 = 645$ meters

2. What is the approximate boiling point of water at sea-level (H=0 meters) according to this equation? Round your answers to the nearest 10 degrees.

The temperature at which water boils at sea level according to the formula may be deduced by setting H = 0 and solving for T:

$1{,}000(100 - T) + 580(100 - T)^2 = 0$

$(100 - T) + .58(100 - T)^2 = 0$

$(100 - T)(1 + .58(100 - T)) = 0$

$(100 - T)(1 + 58 - .58T) = 0$

$(100 - T)(59 - .58T) = 0$

So $T = 100$ or $T = \frac{59}{.58} = \frac{58}{.58} + \frac{1}{.58} \approx 102$

The equation predicts the boiling point is approximately 100°C

Answer Key

24.0: Students use and know simple aspects of a logical argument:

24.1: Students explain the difference between inductive and deductive reasoning and provide examples of each.
24.2: Students identify the hypothesis and conclusion in a logical deducation.
24.3: Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

a. Verify to your own satisfaction, by direct calculation, the correctness of the following equations (do not submit your calculations on this exam):

$$3 = \frac{3}{2}(3^1 - 1)$$

$$3 + 3^2 = \frac{3}{2}(3^2 - 1)$$

$$3 + 3^2 + 3^3 = \frac{3}{2}(3^3 - 1)$$

$$3 + 3^2 + 3^3 + 3^4 = \frac{3}{2}(3^4 - 1)$$

1. Using inductive reasoning, propose a formula that gives the sum for $3 + 3^2 + 3^3 + \ldots + 3^n$ for any counting number n.

$$3 + 3^2 + 3^3 + \ldots + 3^n = \frac{3}{2}(3^n - 1)$$

2. Does the sequence of formulas above prove that your answer to part 1 is correct? Explain your answer.

No, inductive reasoning is really a form of guessing based on previous observations.
[Note to the reader: In this case the formula given in part 1 is correct for any value of n. Inductive reasoning worked in this case, but it doesn't always give correct answers.]

Answer Key

24.0: Students use and know simple aspects of a logical argument:

24.1: Students explain the difference between inductive and deductive reasoning and provide examples of each.
24.2: Students identify the hypothesis and conclusion in a logical deducation.
24.3: Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

[CONTINUED]

b. Consider the following mathematical statement:

If y is a positive integer, then $1 + 1141y^2$ is not a perfect square.

1. Write the hypothesis of this statement.

   y is a positive integer

2. Write the conclusion of this statement.

   $1 + 1141y^2$ is not a perfect square

3. Use whole number arithmetic to prove that the conclusion is correct when $y = 1$.

   If $y = 1$, $1 + 1141y^2 = 1142$. To show that 1142 is not a perfect square, it suffices to show that 1142 falls between the squares of two consecutive integers.

   $33^2 = 1089 < 1142 < 1156 = 34^2$

   Therefore the conclusion, $1 + 1141y^2$ is not a perfect square, is correct when $y = 1$.

Answer Key

24.0: Students use and know simple aspects of a logical argument:

24.1: Students explain the difference between inductive and deductive reasoning and provide examples of each.

24.2: Students identify the hypothesis and conclusion in a logical deducation.

24.3: Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

[CONTINUED]

4. It has been shown by mathematicians that the conclusion is correct for each positive integer y up to and including 30,693,385,322,765,657,197,397,207. However, if this number is increased by 1 so that

$$y = 30{,}693{,}385{,}322{,}765{,}657{,}197{,}397{,}208$$

then the positive square root of $1 + 1141y^2$ is

$$1{,}036{,}782{,}394{,}157{,}223{,}963{,}237{,}125{,}215$$

Is the statement, "If y is a positive integer, then $1 + 1141y^2$ is not a perfect square" correct? Explain your answer.

No, the statement is incorrect because the conclusion is false for at least one positive integer value of y.
[Note that inductive reasoning for this problem would most likely lead to a faulty conclusion.]

Answer Key

25.0: Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:

25.1: Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.
25.2: Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.
25.3: Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.

a. Prove, using basic properties of algebra, or disprove by finding a counterexample, each of the following statements:

1. The set of even numbers is closed under addition.

A number m is even if and only if m = 2k for some integer k. Let m and n be even and let m = 2k and m = 2j for integers k and j. Then:

$$m + n = 2k + 2j = 2(k + j)$$

Therefore m + n has a factor of 2 so it is even. This proves that the sum of any two even numbers is even and therefore the set of even numbers is closed under addition.

2. The sum of any two odd numbers is even.

A number m is odd if and only if m = 2k + 1 for some integer k. Let m and n be odd and let m = 2k +1 and n = 2j + 1 for integers k and j. Then:

$$m + n = (2k + 1) + (2j + 1) = 2k + 2j + 2 = 2(k + j + 1)$$

Therefore m + n has a factor of 2 so it is even. This proves that the sum of any two odd numbers is odd.

Answer Key

24.0: Students use and know simple aspects of a logical argument:

24.1: Students explain the difference between inductive and deductive reasoning and provide examples of each.
24.2: Students identify the hypothesis and conclusion in a logical deducation.
24.3: Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

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a.

3. For any positive real number x, $\sqrt{x} < x$

This statement is false. $x = \frac{1}{4}$ gives a counterexample because:

$\frac{1}{2} = \sqrt{\frac{1}{4}} > \frac{1}{4}$

b. Find all possible pairs of numbers a and b which satisfy the equation $a^2 + b^2 = (a + b)^2$. Explain your reasoning.

Suppose $a^2 + b^2 = (a + b)^2$
Then $a^2 + b^2 = a^2 + 2ab + b^2$
Therefore $0 = 2ab$
Therefore $ab = 0$
Therefore $a = 0$ or $b = 0$.
If $a = 0$ or $b = 0$ then $a^2 + b^2 = (a + b)^2$.
We conclude that $a^2 + b^2 = (a + b)^2$ if and only if $a = 0$ or $b = 0$

Answer Key

24.0: Students use and know simple aspects of a logical argument:

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c. Identify the step below in which a fallacy occurs:

Step 1: Let $a = b = 1$
Step 2: $a^2 = ab$
Step 3: $a^2 - b^2 = ab - b^2$
Step 4: $(a - b)(a + b) = b(a - b)$
Step 5: $a + b = b$
Step 6: $2 = 1$

Answer: Step 5

Explain why the step you have chosen as the fallacy is incorrect.

Step 5 results from dividing both sides of the previous equation by zero because $a - b = 0$.

d. Is the following equation true for some values of x, no values of x or all values of x?

$$16\left(x - \frac{1}{4}\right)^2 - 1 = 16x^2 - 8x$$

The equation is true for all values of x because for any value of x:

$$16\left(x - \frac{1}{4}\right)^2 - 1 = 16\left(x^2 - \frac{1}{2}x + \frac{1}{16}\right) - 1$$

$$= 16x^2 - 8x + 1 - 1$$

$$= 16x^2 - 8x$$